MATHEMATICS

MX25TH.

Full Marks - 80

Time : Three hours

Attempt all questions.

For Question Nos. 1 to 5, write the letter corresponding to the correct answer.

The figures in the right hand margin indicate full marks for the questions.

1. The field property of real numbers stated as "For any real numbers x, y, z, x(y + z) = xy + xz" is expressed by 1
   (A) Multiplication is associative of addition
   (B) Addition is commutative of addition
   (C) Multiplication distributes of addition
   (D) Addition distributes over multiplication

2. If the pair of linear equations 2x + 3y = 7 and 2ax + (a + b)y = 28 has infinitely many solutions, then 1
   (A) a = 2b
   (B) b = 2a
   (C) a + 2b = 0
   (D) 2a + b = 0

3. The next term of the AP : $\sqrt{27}$, $\sqrt{48}$, $\sqrt{75}$, ... is : 1
   (A) $\sqrt{108}$
   (B) $\sqrt{117}$
   (C) $\sqrt{126}$
   (D) $\sqrt{138}$

4. If a pole of 6 m casts a shadow $2\sqrt{3}$ m long on the ground, then the altitude of the sun is : 1
   (A) 30^0
   (B) 60^0
   (C) 45^0
   (D) 90^0

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5. Three metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively are melted to form a single solid sphere. The radius of the resulting sphere is : 1

   (A) 12 cm

   (B) 6 cm

   (C) 36 cm

   (D) 18 cm

6. For what value of k will $5x^4 + 4x^3 + 3x^2 + kx + 4$ contain $x - 1$ as a factor ? 1

7. If $a + b + c = 0$, what is the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$? 1

8. State the nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ when $b^2 - 4ac < 0$. 1

9. When is a number α said to be a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$? 1

10. The length of the sides of a right triangle are 8 cm, 15 cm and 17 cm. State whether the triangle is a right triangle or not. 1

11. In a circle of radius 6.3 cm, an arc AB subtends an angle 90^0 at the centre O of the circle. Find the length of the arc AB. 1

12. How long is the radius of the circle circumscribing a rectangle of sides 8 cm and 6 cm ? 1

13. When do you say that a die is fair ? 1

14. Find any six consecutive composite numbers less than 300. 2

15. Without solving, show that the roots of the quadratic equation $6x^2 + 13x + 6 = 0$ are reciprocal of one another. 2

31. State and prove SAS Similarity Theorem. 6

Or,

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then prove that the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

32. The following is the frequency distribution of the number of teachers in Higher Secondary Schools in 1978 in India. Find the average number of teachers per Higher Secondary School in India for 1978. 6

No. of Teachers	No. of Hr. Sec. Schools
6 - 10	955
11 - 15	1067
16 - 20	1663
21 - 25	1492
26 - 30	1220
31 - 35	1129
36 - 40	745
41 - 45	637
46 - 50	442

(Convert into continuous classes)

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16. Find the 15^{th} term from the last term (towards the first term) of the AP : 3, 7, 11, ... 123. 2

17. Show that :
$\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$ 2

18. A cylindrical container of radius 3 cm and height 14 cm is filled with ice-cream. The ice-cream is to be distributed among 16 children in equal cones with hemispherical tops. If the height of the conical portion is 5 times the radius of its base, find the volume of each ice-cream cone. 2

19. Factorise $x^2 - 6x - 7$ by using Factor Theorem. 3

20. Factorise :
$(a + b + c)^3 - a^3 - b^3 - c^3$. 3

21. Draw the graphs of $4x - 5y + 16 = 0$ and $2x + y - 6 = 0$. Calculate the area bounded by these lines and the X-axis. 3

22. Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank of base 30 m x 22 m. Find the time during which the level of water in the tank rises by 35 cm. 3

23. Show that $\text{cosec}^2\theta + \sec^2\theta$ can never be less than 2. 3

Or,

If θ is an acute angle and $\tan\theta + \cot\theta = 2$, find the value of $\tan^5\theta + \cot^5\theta$.

24. A die is tossed 4 times. Find the probability that 6 appears at least once. 3

25. Show that one of three consecutive odd integers is a multiple of 3. 4

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26. A dealer buys a pocket radio set at a certain sum of money. By selling it at Rs 171 he gains exactly as much percent as the radio set had cost him. Find the price of the radio set. 4

27. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find the coordinates of the centroid. 4

28. Draw a circle of radius 3 cm. Construct a pair of tangents to the circle from a point 5 cm away from the centre. Measure the length of the tangents. Verify your measurement by geometrical reasoning.
(Traces of construction only) 5

29. From the top of a tower 50 m high the angles of depression of the top and the bottom of a pole are observed to be 45^0 and 60^0 respectively. Find the height of the pole, if the pole and the tower stand on the same horizontal plane. (Take $\sqrt{3} = 1.73$) 5

30. A canvas tent is in the form of a cylinder of diameter 16 m and height 5 m surmounted by a cone of equal base and height 6 m. Find the capacity of the tent and the cost of the canvas at Rs 150 per square metre. [use $\pi = 3.14$] 5

Or,

In a circle of radius 6.3 cm, an arc AB subtends an angle of 90^0 at the centre O of the circle. Find

(i) The area of the sector AOB

(ii) The area of the minor segment and the major segment formed by the chord AB.

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